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Pre-service Mathematics Teachers' Perceptions Regarding Methodologies for Enhancing Cognitive Competencies and Use of Educational Resources

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Abstract: The aim of this study is to explore pre-service teachers' perceptions regarding methodologies for enhancing the cognitive activities of future mathematics teachers through the use of educational resources. The study was designed within the framework of mixed research methodology integrating both quantitative and qualitative approaches. The participant group consisted of 80 pre-service mathematics teachers enrolled at various universities in Almaty, Kazakhstan. Data were collected through a semi-structured interview form developed by the researchers, which included both closed-ended items to capture participants' self-reported cognitive awareness, cognitive flexibility, and mathematical thinking levels, and open-ended questions to explore their perceptions and experiences. The findings indicate that the cognitive awareness levels of the participating pre-service mathematics teachers are at a moderate level. Similarly, their levels of cognitive flexibility were found to be moderate. In contrast, the participants demonstrated a high level of mathematical thinking. Despite this strength, the majority of participants reported that they perceive themselves as only partially competent in terms of utilizing methodologies aimed at improving their cognitive activities through educational resources. Based on these findings, it is concluded that there is a need for systematic improvement in teacher education programs. In particular, curricula in faculties of education should be revised and restructured to better integrate methodologies that support the development of cognitive activity through effective use of educational resources.

Keywords: *Cognitive activity, cognitive awareness, cognitive flexibility, mathematical thinking, pre-service mathematics teachers*

Introduction

Contemporary approaches to learning place learning, and so the learner, at the center of educational processes. Within this perspective, learners are expected to develop an awareness of their own competences, abilities, planning skills, thinking processes, and behaviors, as well as to regulate and control their cognitive processes (Uzunboylu, Tezer and Yildiz, 2020). In this context, learning to learn competencies—often conceptualized under the umbrella of cognitive awareness—have become indispensable in today's society. These competencies encompass a broad range of higher-order skills, including the ability to monitor

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one's own learning progress, articulate the rationale behind actions, reflect on affective states, engage in planning and problem-solving, and exercise self-regulation and cognitive control. Such skills are increasingly recognized as foundation for fostering autonomous and reflective learners (Sener and Mede, 2023).

Conceptual framework

Research in education indicate that improving students' research, inquiry and higher-order thinking abilities is an essential mission of educational institutions (Antonio and Prudente, 2024). Cognitive development is widely recognized as a fundamental component of effective education. In contemporary teacher education programs, fostering pre-service teachers' cognitive activity is essential not only for their own professional growth but also for the quality of instruction they will provide to future learners (Gürsan, Tapan-Broutin and Ipek, 2025). The capacity to reflect on one's thinking, adapt flexibly to new situations, and employ structured reasoning are all critical skills for pre-service teachers (Schoenfeld, 2016). Cognitive activity involves the active construction of mental schemes (Ovadiya, 2025). For pre-service teachers, this activity is twofold: they must engage with the content as learners and simultaneously analyze the cognitive demands that various educational resources place upon their future students.

Therefore, it is crucial for pre-service teachers to not only build strong knowledge of their subject matter and teaching methods but also to develop advanced cognitive abilities that will aid in their professional and personal development (Christodoulou and Papanikolaou, 2023). Among these, cognitive flexibility plays a crucial role, as it enables individuals to adapt their thinking to new situations, perspectives, and problem contexts (Abdellah, 2015). Cognitive flexibility is defined as the mental ability to switch between different concepts or to adapt one's thinking to new, unforeseen situational requirements (Spiro et al., 2012). In mathematics education, this manifests as the ability to approach a single problem through multiple representations—algebraic, geometric, and numerical (Strayer, 2012). Pre-service teachers with high perceived cognitive flexibility are better equipped to navigate "non-routine" problems and are more likely to adopt student-centered methodologies that allow for diverse solution paths rather than rigid, rule-based instruction. Furthermore, teacher self-efficacy is recognized as a key factor influencing the successful enactment of teaching practices (Wyatt, 2014).

Teaching itself constitutes a cognitively, socially, and emotionally demanding professional practice, involving continuous processes of thinking, monitoring, planning, and evaluation. In order to effectively support teachers' professional development, it is necessary to establish learning environments that foster not only pedagogical competence but also cognitive development (Jennings and Greenberg, 2009). In this regard, the deliberate use of cognitive processes—often framed as thinking about thinking—across different stages of instructional practice contributes significantly to the effectiveness of teaching. Indeed, teachers' capacity to monitor and evaluate classroom interactions, particularly the dynamic relationship between teacher and student, is closely linked to their cognitive awareness (Saricam and Ogurlu, 2015).

Cognitive awareness, often associated with metacognitive functioning, can be defined as an individual's capacity to rapidly evaluate the significance of a given situation, concept, or event (Broom, 2010). This capacity supports the regulation of higher-order thinking processes, including planning, monitoring, and evaluating problem-solving strategies. Moreover, it enables individuals to detect, correct, and reflect upon errors during the learning process (Menlah and Boateng, 2025). Within educational contexts, the quality of teacher–student interactions plays a pivotal role in determining not only what is taught, but also how, when, and to whom instruction is delivered. In this sense, cognitive awareness facilitates learners' ability to organize their knowledge structures and develop a deeper understanding of their own learning processes (Klassen and Chiu, 2011). Schraw and Dennison (1994) argue that high levels of cognitive awareness allow individuals to plan, sequence, and monitor their problem-solving strategies. For pre-service teachers, perceiving one's own cognitive awareness is a prerequisite for developing the "professional noticing" required to identify and address student misconceptions (Sherin and van Es, 2009)

Related research

A review of the relevant literature indicates that cognitive awareness in teacher education is a multifaceted construct associated with a wide range of variables. Empirical studies have demonstrated significant relationships between cognitive awareness and academic achievement (Abdellah, 2015), professional experience (Ates and Ceran, 2020), reading proficiency (Negretti and Kuteeva, 2011), cognitive flexibility (Yurt and Saracaloglu, 2026), and self-regulation (Whitebread and O'Sullivan, 2012). Within the dynamic and complex nature of learning processes, both cognitive awareness and critical thinking are increasingly emphasized in teacher education programs as essential components for developing meaningful learning and robust reasoning skills among pre-service teachers (Lee, Teo and Chai, 2010). Cognitive awareness is reported as a critical predictor of problem-solving success and pedagogical efficacy. Research indicates that pre-service teachers who possess high levels of cognitive awareness are better equipped to monitor their own understanding and regulate their instructional strategies (Wilson and Clarke, 2004). Desoete (2009) argues that cognitive "off-line" processes—prediction, planning, monitoring, and evaluation—are essential for pre-service teachers to bridge the gap between theoretical knowledge and classroom practice. However, Kuzle (2017) suggests that while pre-service teachers may recognize the importance of cognitive awareness, they often struggle to implement these processes when faced with complex mathematical tasks, indicating a discrepancy between theoretical awareness and practical application.

On the other hand, cognitive flexibility allows teachers to represent mathematical concepts in multiple ways and to adapt their instruction to the diverse needs of students. According to Eraky, Leikin and Hadad (2025), cognitive flexibility is particularly vital in domains like mathematics education. In the realm of teacher education, Silver (1997) identifies flexibility as a core component of mathematical creativity, involving the ability to switch between different solution strategies. Kusci and Yurt (2026) emphasize that pre-service teachers often exhibit cognitive rigidity due to their own experiences with traditional, rote-based learning, which poses a significant challenge for modern teacher education programs that emphasize adaptive expertise. Several empirical studies provide further insight into these relationships. For instance, Clipa, Ignat and Rusu (2011) investigated the interrelations among pre-service teachers' self-assessment, learning motivation, and metacognitive levels in a sample of 92 students enrolled in the Department of Educational Sciences at Suceava University. Their findings revealed a statistically significant and positive relationship between metacognitive abilities and self-evaluation levels. Moreover, a strong positive association was identified between self-evaluation and academic performance, underscoring the role of metacognitive processes in academic success. In a related study, Halamish (2018) examined the relationship between learning strategies and cognitive knowledge among 83 in-service teachers and 82 pre-service teachers. The findings indicated that the use of learning strategies did not lead to significant differences in participants' cognitive knowledge levels. Furthermore, the study reported that teachers' knowledge of learning strategies did not substantially increase with years of professional experience, suggesting that experience alone may not be sufficient for the development of metacognitive competence. Braund and Soleas (2019) explored teachers' and pre-service teachers' cognitive conceptualizations, practices, and beliefs through a mixed-methods design involving 43 teachers and 45 teacher candidates. The results demonstrated that both groups experienced difficulties in effectively implementing metacognitive processes in instructional contexts. While in-service teachers exhibited more practical, context-sensitive, and creative forms of metacognitive adaptation in classroom settings, pre-service teachers were found to require additional support in developing coherent conceptual understanding and in operationalizing metacognitive knowledge. Focusing on instructional design, Kwanjai and Sumalee (2012) examined the development of a constructivist learning environment model aimed at enhancing cognitive flexibility in higher education. Their findings suggested that the proposed model was theoretically coherent and aligned with established instructional design principles. Importantly, students who engaged with this model demonstrated higher levels of cognitive flexibility and academic achievement, highlighting the potential of constructivist approaches to support advanced cognitive development.

Recent literature has increasingly focused on the intersection of cognitive competencies and the pedagogical integration of modern educational resources. For instance, Birgili et al. (2025) highlighted a critical theory-to-practice gap, revealing that while pre-service mathematics teachers may exhibit high theoretical awareness of cognitive strategies, they frequently experience cognitive overload when attempting to embed these strategies within resource-rich lesson plans. Furthermore, a study by Depaepe et al. (2015) corroborated that pre-service teachers struggle significantly with the metacognitive evaluation phase of instructional design, often failing to anticipate how specific educational resources will alter the cognitive demand of a mathematical task. These findings collectively argue that modern teacher education programs must move beyond isolated cognitive training, instead embedding cognitive awareness and flexibility development directly within the context of resource-integrated pedagogical design.

Moreover, the rapid proliferation of artificial intelligence and advanced algorithmic tools in educational settings has introduced a new layer of complexity to pre-service teachers' cognitive activities. Bernardi et al. (2025) highlights that while pre-service mathematics teachers are increasingly exposed to generative digital resources, their cognitive regulation often lags their technical fluency. When designing tasks using these advanced educational resources, prospective teachers frequently experience a pedagogical disconnect, where their high personal mathematical thinking does not successfully translate into cognitively demanding task design for students. This underscores a critical need to understand how the integration of next-generation digital resources impacts the self-efficacy and cognitive awareness of future educators, shifting the focus from mere tool utilization to cognitive orchestration.

Furthermore, the demand for cognitive flexibility has been amplified by the widespread adoption of interactive and collaborative mathematics platforms. A study by Ochogboju and Diez-Palomar (2025) within the context of technology-mediated learning environments revealed that pre-service teachers often struggle with representational flexibility during real-time, technology-mediated instruction. Although participants in their study demonstrated a strong theoretical grasp of multiple mathematical representations, they exhibited representational rigidity when forced to adapt their instructional strategies spontaneously in response to student interactions within digital spaces. This finding suggests that modern educational resources, rather than naturally fostering cognitive flexibility, may actually expose underlying deficits in pre-service teachers' adaptive pedagogical expertise.

The intersection of cognitive monitoring and resource utilization has also been a focal point of recent empirical investigations. Investigating the use of immersive and gamified mathematics resources, Dündar and Öztürk (2026) observed that pre-service teachers often over-rely on the external scaffolding provided by the software, thereby diminishing their internal cognitive regulation. Their research indicated that the rich informational feedback embedded in modern educational resources can induce cognitive overload, making it exceedingly difficult for novice teachers to critically evaluate whether the mathematical learning objectives are being authentically met. Consequently, a pre-service teacher's perceived competence in using these resources may sometimes be an illusion, masking a deeper inability to orchestrate meaningful conceptual development.

More recently, the concept of pedagogical friction has emerged as a crucial lens for examining the gap between abstract mathematical thinking and practical instructional implementation. As articulated by Salsabillah, Wagyina and Rohmah (2026), pre-service mathematics teachers frequently construct a highly idealized mathematical identity, perceiving themselves as creative and proficient problem solvers. However, when required to deploy complex educational resources to mediate student learning, this idealized self-perception often clashes with practical instructional realities, resulting in a sharp decline in pedagogical self-efficacy. Their findings demonstrate that high mathematical thinking alone is insufficient; without targeted training in the cognitive and pedagogical orchestration of educational tools, pre-service teachers remain conceptually strong but pedagogically constrained.

As highlighted by Weigand, Trgalova and Tabach (2024), modern teacher preparation must evolve beyond the mere introduction of educational technologies and resources. Instead, the instructional focus must pivot towards explicitly cultivating the intersectional domain of cognitive flexibility and resource-based pedagogical content knowledge. Understanding how pre-service teachers perceive their cognitive competencies in this rapidly evolving, resource-rich educational landscape is no longer just a theoretical inquiry; it is a vital prerequisite for designing interventions that transform theoretical mathematical knowledge into adaptive, highly effective teaching practices.

The literature suggests that while cognitive awareness and related constructions such as cognitive flexibility and mathematical thinking are widely recognized as critical components of teacher education, there remain notable challenges in their effective development and implementation. In particular, translating these cognitive competencies into practice can be challenging, suggesting that pre-service teachers could benefit from further structured and systematic support.

The final strand of literature explores how pre-service teachers perceive their own cognitive competencies. Bandura's (2013) social cognitive theory posits that self-efficacy—the belief in one's ability to succeed—is a primary driver of behavior. In mathematics education, Charalambous (2015) found a strong correlation between a teacher's self-perception of their cognitive abilities and the cognitive level of the instruction they provide. If pre-service teachers perceive themselves as lacking in cognitive flexibility or awareness, they are less likely to employ innovative methodologies that require these traits. Thus, understanding these perceptions is vital for designing teacher education curricula that not only provide tools but also build the internal confidence necessary to use them effectively. Indeed, following the accelerated digitalization of learning environments, the conceptualization of cognitive activity has expanded to include the orchestration of complex educational resources. Recent research emphasize that pre-service mathematics teachers must develop "adaptive pedagogical expertise" to navigate technology-rich and resource-diverse environments (Engelbrecht, Llinares and Borba, 2020). The integration of dynamic geometry software, virtual manipulatives, and advanced modeling resources demands a higher threshold of cognitive flexibility and metacognitive monitoring than traditional instructional methods. Consequently, the ability to select, evaluate, and pedagogically integrate these resources is no longer a peripheral skill but a core component of mathematical knowledge for teaching (Stoilescu, 2015). This paradigm shift underscores the urgent need to understand how prospective teachers perceive their cognitive readiness to mediate student learning through these advanced educational tools.

Purpose of the research

The primary aim of this study is to examine pre-service mathematics teachers' perceptions of methodologies aimed at enhancing their cognitive activity through the use of educational resources. In particular, the study seeks to explore how pre-service teachers evaluate their own cognitive competencies and their preparedness to engage with pedagogical approaches that foster cognitive development.

In line with this overarching aim, the study is guided by the following research questions:

1. How do pre-service mathematics teachers perceive their levels of cognitive awareness?
2. How do pre-service mathematics teachers perceive their levels of cognitive flexibility?
3. How do pre-service mathematics teachers perceive their levels of mathematical thinking?
4. To what extent do pre-service mathematics teachers perceive themselves as competent in employing methodologies aimed at enhancing their cognitive activities through educational resources?

Materials and methods

Research design

This study was designed within the framework of a mixed-methods approach, integrating both quantitative and qualitative data to provide a comprehensive understanding of pre-service mathematics teachers' cognitive competencies and their use of educational resources. Specifically, a convergent parallel mixed-methods design was adopted, in which quantitative and qualitative data were collected concurrently, analyzed separately, and then integrated during the interpretation phase.

The quantitative component of the study aimed to identify general trends in participants' self-reported cognitive awareness, cognitive flexibility, and mathematical thinking levels through descriptive statistical analysis (e.g., frequency, percentage, and mean scores). This strand enabled the researchers to capture the overall distribution and tendencies within the participant group.

Complementing this, the qualitative component focused on exploring participants' in-depth perspectives regarding their competencies in utilizing educational resources to enhance cognitive activities. Qualitative data obtained from open-ended responses were analyzed through categorization and thematic interpretation, allowing for a nuanced understanding of participants' experiences and perceived challenges. Employing a mixed-methods design allowed for the triangulation of data and enhanced the validity of the findings (Creswell and Poth, 2016). While quantitative data provide a broad overview of cognitive competency levels, the qualitative findings offer explanatory depth by revealing how and why participants perceive themselves as competent or insufficient in specific areas.

Participants

The participants of the study consisted of 80 pre-service mathematics teachers enrolled in the Departments of Mathematics Education at various universities in Almaty, Kazakhstan, during the 2021–2022 academic year. Participation in the study was voluntary, and all participants were informed about the purpose of the research prior to data collection. Table 1 shows the demographic characteristics of the participants.

Table 1. *Demographic Characteristics of Participants*

Variable	Category	Frequency (f)	Percentage (%)
Gender	Female	33	41.25%
	Male	47	58.75%
Academic Year	1st Year	11	13.75%
	2nd Year	16	20.00%
	3rd Year	20	25.00%
	4th Year	33	41.25%
Total		80	100%

Of the participant pre-service mathematics teachers, 33 were female and 47 were male. In terms of academic level, 11 participants were first-year students, 16 were second-year students, 20 were third-year students, and 33 were fourth-year students. This distribution allowed for the inclusion of perspectives from different stages of teacher education, thereby enriching the dataset.

Data collection tools

The data for this study were collected through a semi-structured interview form developed by the researchers. During the development process, a comprehensive review of relevant literature was conducted to ensure conceptual alignment with existing studies. Based on this review, a preliminary version of the interview form was constructed to elicit pre-service mathematics teachers' perspectives on methodologies aimed at enhancing their cognitive activities through educational resources. To establish content validity, the draft interview form was submitted to three experts in the field of mathematics education and educational sciences. Following their feedback, necessary revisions were made to improve the clarity, relevance, and representativeness of the items.

Subsequently, a pilot implementation was conducted with two pre-service mathematics teachers to assess the comprehensibility and functionality of the instrument. Based on the positive feedback obtained, the interview form was finalized for data collection. The participants involved in the pilot study were excluded from the main study sample. The final version of the semi-structured interview form is presented in the Appendix.

Data collection process

Due to logistical considerations, the data collection process was conducted through electronic communication. Initially, pre-service mathematics teachers were contacted via email and informed about the purpose and scope of the study. Those who voluntarily agreed to participate were sent the semi-structured interview form electronically. Participants were asked to complete the interview form and return it to the researchers via email. The entire data collection process, including the completion and submission of the interview forms, was carried out over a period of approximately one month. This procedure ensured flexibility and allowed participants sufficient time to provide thoughtful and reflective responses (Brinkmann and Kvale, 2015), thereby enhancing the richness of the qualitative data.

Data collection analysis

The data obtained from the semi-structured interview forms were analyzed using a combination of descriptive and content analysis techniques. Responses to closed-ended items were analyzed quantitatively through frequency, percentage, and mean score calculations, allowing for the identification of general trends in participants' responses.

For Likert-type items, a three-point scale was employed, where "Always" was assigned 3 points, "Sometimes" 2 points, and "Never" 1 point. The interpretation intervals for mean scores were determined as follows: 2.34–3.00 (Always), 1.67–2.33 (Sometimes), and 1.00–1.66 (Never). These intervals facilitated the systematic interpretation of participants' responses.

Responses to open-ended questions were analyzed qualitatively through categorization and thematic grouping. Codes were generated to identify recurring patterns in the participants' perceptions of cognitive methodologies. Frequencies and percentages were also calculated to support the interpretation of qualitative findings. In addition, direct quotations from participants were included to enhance the credibility and richness of the data. To ensure confidentiality, participants were anonymized using codes (e.g., TC-1, TC-2, TC-3).

Validity and Reliability

In order to ensure the methodological rigor of the study, several strategies were employed to enhance the validity and reliability of both the quantitative and qualitative components within the mixed-methods framework. For the quantitative strand, internal consistency was supported through the use of clearly defined Likert-type items and standardized scoring procedures. The interpretation intervals for mean scores were determined a priori to ensure consistency in data interpretation. The use of descriptive statistics (frequency, percentage, and mean) contributed to the transparency and replicability of the findings.

For the qualitative strand, measures have been taken to ensure the validity and reliability of the research (Creswell and Poth, 2016). To enhance credibility, the data collection instrument was reviewed by experts and piloted prior to implementation. Direct quotations from participants were included to ensure that findings accurately reflected participants' perspectives. To strengthen the reliability of qualitative analysis, an inter-coder agreement procedure was employed. A second researcher independently coded a subset of the data, and the level of agreement between coders was calculated using the formula proposed by Miles and Huberman (1994): agreement / (agreement + disagreement). The resulting agreement rate exceeded the commonly accepted threshold of 80%, indicating a satisfactory level of coding reliability. A systematic and transparent data analysis process, including clear documentation of coding procedures and category development was undertaken.

Findings and Interpretations

This section presents the findings obtained from both the quantitative and qualitative components of the study in an integrated manner. The findings are organized around the main variables of the research, namely cognitive awareness, cognitive flexibility, mathematical thinking, and competencies in the use of educational resources.

Pre-service Mathematics Teachers' Perceptions of Their Cognitive Awareness

The first research question investigated how pre-service mathematics teachers perceive their own levels of cognitive awareness during the learning process. The quantitative distribution of the participants' responses, including frequencies (f), percentages (%), and mean scores (X), is presented in Table 2.

Table 2. *Descriptive Analyses of Pre-service Teachers' Perceptions of Cognitive Awareness*

	Item	Always		Sometimes		Never		Total		Mean
		f	%	*f*	%	*f*	%	*f*	%	X
1.	While listening to the lecture, I question whether I understand	24	30	42	52.5	14	17.5	80	100	2.12
2.	I try to understand the purpose of the subject I will learn in the lesson.	32	40	38	47.5	10	12.5	80	100	2.27
3.	When I encounter a problem related to a subject that I do not know in the lesson, I anticipate possible solutions about that subject.	34	42.5	39	48.75	7	8.75	80	100	2.33
4.	Getting support from the teacher about the subject of the lesson helps me to be a successful student.	13	16.25	51	63.75	16	20	80	100	1.96
5.	I can identify critical concepts of the subject of the lesson	19	23.75	33	41.25	28	35	80	100	1.65
6.	I can evaluate myself whether the course has achieved its objectives or not.	9	11.25	41	51.25	30	37.5	80	100	1.73
	Composite Mean Score									2.01

Note: Interpretation intervals: 1.00–1.66 (Low/Never); 1.67–2.33 (Moderate/Sometimes); 2.34–3.00 (High/Always)

As illustrated in Table 2, the composite mean score for cognitive awareness is $\bar{X} = 2.01$, which falls within the "Moderate" range according to the predefined interpretation intervals. This suggests that while pre-service teachers possess a foundational level of awareness regarding their mental processes, this awareness is inconsistent and varies significantly across different cognitive dimensions.

The findings indicate that participants perceive themselves as most competent in the predictive and orienting phases of cognitive activity. The highest level of agreement was observed for Item 3, regarding the anticipation of solutions for unfamiliar problems (X = 2.33). This is closely followed by the effort to understand the purpose of the subject matter (Item 2, X = 2.27). These findings suggest that pre-service teachers are inclined toward "forward-looking" cognitive strategies, prioritizing the goal-setting and preparatory stages of mathematical engagement.

In contrast, a significant decline in perceived competence was observed in items related to higher-order metacognitive regulation and self-evaluation. Specifically, the ability to identify critical conceptual structures (Item 5) yielded the lowest mean score (X = 1.65), bordering on the "Low" perception interval. Furthermore, the capacity for self-evaluation regarding the attainment of learning objectives (Item 6) was also notably low (X = 1.73). These findings reveal a critical pedagogical gap: while pre-service teachers are moderately active in "monitoring" their listening (Item 1, X = 2.12), they struggle to "evaluate" the outcomes of that cognitive effort. This suggests that their cognitive awareness is more procedural than structural; they can engage in the act of learning but find it difficult to identify the core mathematical essence or determine the success of their own cognitive trajectories. The moderate reliance on external support (Item 4, X = 1.96) further reinforces this conclusion, suggesting that participants may lack the necessary internal metacognitive scaffolds to achieve independent conceptual mastery, often looking to the instructor to bridge the gap in their own cognitive awareness.

Pre-service Mathematics Teachers' Perceptions of Their Cognitive Flexibility

The second research question aimed to determine the perceived levels of cognitive flexibility among pre-service mathematics teachers, specifically focusing on their ability to adapt their thinking and behaviors to varied pedagogical and mathematical contexts. Table 3 presents the distribution of pre-service mathematics teachers' responses regarding their levels of cognitive flexibility.

Table 3. *Descriptive Analyses of Pre-service Teachers' Perceptions of Cognitive Flexibility*

	Item	Always		Sometimes		Never		Total		Mean
		f	%	*f*	%	*f*	%	*f*	%	*X*
1.	I can express an idea in many different ways	26	32.5	30	37.5	24	35	80	100	2.02
2.	I can find useful solutions to problems that seem unsolvable.	19	23.75	43	53.75	18	22.5	80	100	2.01
3.	I am willing to produce creative solutions to problems.	23	28.75	51	63.75	6	7.5	80	100	2.21
4.	I am confident enough to try different behavior patterns.	36	45	39	48.75	5	6.25	80	100	2.38
5.	In any situation that I encounter, I can act appropriately.	49	61.25	28	35	3	3.75	80	100	2.57
	Composite Mean Score									2.24

Note: Interpretation intervals: 1.00–1.66 (Low); 1.67–2.33 (Moderate); 2.34–3.00 (High)

The findings presented in Table 3 indicate that the overall perceived cognitive flexibility of pre-service teachers stands at a moderate to high level (X = 2.24). However, a granular analysis reveals a bifurcated perception: participants report high confidence in situational and behavioral adaptation, yet display comparative hesitation regarding deep cognitive and representational flexibility.

The data reveals that pre-service teachers possess a high degree of self-efficacy regarding their general adaptability. The highest mean scores were recorded for "acting appropriately in encountered situations" (Item 5, X = 2.57) and the "confidence to try different behavior patterns" (Item 4, X = 2.38). These scores suggest that the participants perceive themselves as socially and professionally agile, a trait often associated with the affective domain of professional identity. This perceived behavioral flexibility is a vital precursor to classroom management and instructional responsiveness.

In contrast to their high behavioral confidence, participants reported lower scores in dimensions directly related to mathematical cognition. Specifically, the ability to express an idea in multiple ways (Item 1, X = 2.02) and finding solutions to seemingly unsolvable problems (Item 2, X = 2.01) remained at the lower end of the moderate range. While 61.3% of participants "Always" feel they can act appropriately in a general sense, only 32.5% feel they can "Always" express mathematical ideas in diverse ways. This suggests that while pre-service teachers are willing to be creative (Item 3, X = 2.21), they may lack the specific cognitive tools or "adaptive expertise" necessary to translate this willingness into multiple mathematical representations.

Indeed, the cognitive flexibility of pre-service teachers appears to be more "situational" than "conceptual." Their high perceived ability to adapt behaviorally indicates a readiness for the social demands of teaching; however, the moderate scores in representational flexibility highlight a potential vulnerability. Without the ability to pivot between different mathematical explanations and solution strategies, these pre-service teachers may struggle to support students who require diverse pedagogical approaches to grasp complex concepts.

Pre-service Mathematics Teachers' Perceptions of Their Mathematical Thinking

The third research question explored how pre-service teachers perceive their mathematical thinking skills, with a focus on creativity, connectivity, and the application of non-routine strategies. The descriptive analyses of pre-service teachers' perceptions of mathematical thinking are presented in Table 4.

Table 4. *Descriptive Analyses of Pre-service Teachers' Perceptions of Mathematical Thinking*

	Item	Always		Sometimes		Never		Total		Mean
		F	%	*F*	%	*F*	%	*F*	%	X
1.	My mathematical thinking skills are used in solving daily life problems.	38	47.5	30	37.5	12	15	80	100	2.32
2.	A good mathematician is one who has a high level of creative thinking.	47	58.75	29	36.25	4	5	80	100	2.53
3.	I solve problems in unconventional ways with my mathematical thinking skills	35	43.75	39	48.75	8	10	80	100	2.38
4.	With my mathematical thinking skills, I can connect with my old knowledge while constructing new knowledge.	44	55	33	41.25	3	3.75	80	100	2.51
	Composite Mean Score									2.43

Note: Interpretation intervals: 1.00–1.66 (Low); 1.67–2.33 (Moderate); 2.34–3.00 (High)

The findings indicate that pre-service mathematics teachers possess a high level of perceived mathematical thinking (X = 2.43). Unlike the previous dimensions of awareness and flexibility, which were perceived as moderate, participants appear more confident in their identity as mathematical thinkers and their ability to integrate complex cognitive processes.

The highest mean scores were associated with the broader, more philosophical aspects of mathematics. Participants strongly identified with the notion that mathematical excellence is intrinsically linked to creative thinking (Item 2, X = 2.53). This suggests a move away from the traditional view of mathematics as a purely algorithmic discipline. Furthermore, the ability to bridge prior and new knowledge (Item 4, X = 2.51) was perceived as a significant strength. According to Ausubel's (1968) meaningful learning theory, this connection-making is the cornerstone of conceptual development, and the high scores here indicate that pre-service teachers value and perceive themselves as active constructors of mathematical meaning.

In terms of application, participants reported a high tendency toward using unconventional methods (Item 3, X = 2.38), which aligns with the "specializing and generalizing" processes described by Mason et al. (2010). However, the application of mathematical thinking to daily life problems (Item 1, X = 2.32) received the lowest score within this high-performing category. This suggests a slight "abstraction gap"—while participants feel competent in the theoretical and creative realms of mathematical construction, they find it slightly more challenging to externalize these skills into concrete, real-world contexts.

Pre-service Teachers' Perceptions of Competence in Enhancing Cognitive Activity via Educational Resources

The final research question examined the extent to which pre-service mathematics teachers perceive themselves as competent in employing specific methodologies and educational resources to foster cognitive activity. Table 5 presents the distribution of the competencies of pre-service mathematics teachers participating in the research regarding the methodologies of improving their cognitive activities through educational resources.

Table 5. *Perceived Competencies of Pre-service Mathematics Teachers Regarding Methodologies to Improve Cognitive Activities Through Educational Resources*

Category	Representative Qualitative Excerpts	*f*	%
Very sufficient	TC-4: I find myself very competent in this regard. I think I am very competent in using educational resources in the digital age and in the online education process. TC-48: When I think of educational resources, I first think of additional resources and the internet. I am very proficient in benefiting from additional resources in terms of cognitive activities. TC-61: I am very proficient in cognitive activities, especially using mobile devices.	5	6.25
Sufficient	TC-9: I find myself sufficient. I can use different methods, techniques and materials for cognitive activities. TC-27: I consider myself sufficient to benefit from cognitive activities, especially from educational resources to improve problem-solving skills. TC-77: While solving a problem, I am proficient in using educational resources to explore different solutions and more advanced problems.	17	21.25
Partially sufficient	TC-33: Although the training we received aims to give us competence in this field, I find myself partially sufficient. TC-40: I think I am partially competent because I do not have enough motivation to develop cognitive activities. TC-80: I think that I need to improve myself in using educational resources. Therefore, I find myself partially sufficient.	44	55
Insufficient	TC-1: I think I am inadequate. This is because I do not have enough information to improve cognitive activities. TC-23: I don't think I have any qualifications in this field yet. TC-62: Although I try to improve myself, there are many resources to benefit from in terms of cognitive activities. It is not possible to take advantage of the lack of time. That's why I think I'm inadequate.	11	13.75
Very insufficient	TC-10: I am very inadequate. Because I spend very little time to improve myself outside of classroom activities. TC-55: I am very inadequate in benefiting from educational resources for the development of cognitive activities. TC-59: Even though there are courses on cognitive activities in the training given to us, I am very inadequate in using the educational resources effectively.	3	3.75
Total		80	100

Table 5 reveals a stark contrast to the participants' high self-perceptions of mathematical thinking. A substantial majority of pre-service teachers (72.5%) categorize themselves as only "Partially Sufficient," "Insufficient," or "Very Insufficient." This suggests a significant theory-to-practice gap in mathematics teacher education.

The largest cohort of participants (55%) identified as "Partially Sufficient." The qualitative data suggests that this perception is rooted in an awareness of their own limitations rather than a lack of exposure. For instance, TC-33 noted that while their formal training aimed for competence, the internal sense of mastery remains incomplete. This category represents a state of "conscious incompetence," where students recognize the importance of cognitive resources (such as digital tools or non-routine problems) but lack the Pedagogical Content Knowledge to implement them effectively.

A small but notable group (6.25%) felt "Very Sufficient," primarily citing their fluency with digital and mobile technologies (TC-4, TC-61). However, the qualitative responses from the "Insufficient" group (13.75%) highlight a critical barrier: Information Overload and Time Constraints. As TC-62 observed, the sheer volume of available resources, coupled with a lack of time, renders them unable to benefit from these tools. This indicates that digital literacy alone does not equate to pedagogical competence in fostering cognitive activity.

The findings identify two primary inhibitors of perceived competence which are metacognitive knowledge gaps and affective barriers. Participants like TC-1 explicitly mentioned a lack of "information" on how to specifically improve cognitive activities, despite having mathematical knowledge, interpreted as metacognitive knowledge gaps. For the affective barriers, for example TC-40 attributed their partial competence to a lack of "motivation," suggesting that the drive to move beyond traditional teaching methods is a prerequisite for developing these advanced instructional skills.

Indeed, while pre-service teachers perceive themselves as strong mathematical thinkers, they feel largely unprepared to translate that thinking into classroom practice through educational resources. The dominance of the "Partially Sufficient" category indicates that teacher education programs may be successful in introducing the concept of cognitive enhancement but are less successful in providing the procedural fluency required for confident implementation.

Discussions

The findings of this study indicate that pre-service mathematics teachers exhibit moderate levels of cognitive awareness and moderate levels of cognitive flexibility. These findings are consistent with previous research showing a positive relationship between cognitive awareness and cognitive flexibility among teacher candidates (Hermita and Thamrin, 2015; Yurt and Saracaloglu, 2026). The studies in the literature report pre-service teachers' levels of cognitive awareness and flexibility as moderate or high. For example, Ghonsooly, Khajavy and Mahjoobi (2014) found no significant gender differences in cognitive awareness levels, while Abdellah (2015) identified a positive relationship between cognitive awareness and academic achievement, suggesting that higher cognitive awareness is associated with improved academic performance.

Regarding mathematical thinking, the findings show that participants demonstrate a high level perception of mathematical thinking. This aligns with prior studies reporting a significant relationship between mathematical thinking and problem-solving skills among teachers and teacher candidates (Cai, 2002; Nunokawa, 2005). Yorulmaz, Cokcaliskan and Celik (2018) found that pre-service primary school teachers exhibited high levels of mathematical thinking, moderate individual innovativeness, and pronounced inquisitiveness within the innovativeness dimensions. Conversely, other studies have reported weaker mathematical thinking skills among students and pre-service teachers (Mubark, 2012; Yesildere, 2007). Song and Ginsburg (1987) emphasized that the development of mathematical thinking is influenced by environmental and cultural factors, including classroom practices, teacher attitudes, parental expectations, and educational support.

An important finding of this study is the relatively high level of mathematical thinking, which appears to be inconsistent with the moderate levels of cognitive awareness and flexibility. This difference suggests that mathematical thinking, as perceived by pre-service teachers, may be more closely associated with procedural fluency and problem-solving performance rather than with reflective and cognitive dimensions of thinking. In other words, participants may perceive themselves as competent in "doing mathematics" without necessarily engaging in deeper processes of "thinking about their thinking". The dissonance between high perceived mathematical thinking and moderate cognitive awareness can also be interpreted through the lens of recent technological dependencies in mathematics education. As highlighted by Bernardi et al. (2025), pre-service teachers operating in highly digitalized, computationally rich environments often experience an illusion of mathematical competence. Because advanced computational tools and dynamic software seamlessly handle procedural heavy lifting and immediate visualization, prospective teachers may artificially inflate their self-reported mathematical thinking. However, this technologically mediated fluency frequently bypasses the productive tasks necessary for deep cognitive monitoring, explaining why their foundational cognitive awareness remains merely moderate.

Furthermore, the majority of pre-service mathematics teachers in the present study reported being partially competent in methodologies for enhancing their cognitive activities through educational resources. This finding resonates with Ozdemir's (2008) research, which highlighted that pre-service teachers often encounter difficulties in effectively integrating educational materials and resources to develop mathematical skills. Salsabillah, Wagyina and Rohmah (2026) suggest that when pre-service teachers evaluate their own mathematical thinking, they inherently rely on traditional, algorithmic benchmarks rather than their capacity for adaptive, resource-integrated problem-solving. This compartmentalized view—where mathematical thinking is divorced from pedagogical flexibility—provides a crucial explanation for why participants feel confident in their subject-matter knowledge yet exhibit representational rigidity (Ochogboju and Diez-Palomar, 2025) when required to translate this mathematical knowledge across diverse digital and interactive educational mediums. From a broader perspective, this finding points to a critical gap between access to educational resources and the ability to use these resources in cognitively productive ways. It suggests that exposure to educational technologies or materials alone is insufficient; rather, what is required is a structured pedagogical framework that explicitly links resource use with the development of cognitive awareness and cognitive flexibility competencies. While some studies noted basic difficulties in material integration, Dündar and Öztürk (2026) reveals a modernized form of pedagogical friction. Pre-service teachers today are overwhelmed by the cognitive load required to evaluate, select, and orchestrate highly interactive or AI-driven educational resources. Their self-admitted partial competence is not necessarily a reflection of technological illiteracy, but rather a deficit in pedagogical cognitive flexibility—the specific capacity to dynamically align digital resource affordances with students' emergent cognitive needs during instruction.

Conclusion

This study set out to examine pre-service mathematics teachers' cognitive competencies—namely cognitive awareness, cognitive flexibility, and mathematical thinking—and their perceived competence in utilizing educational resources to enhance these cognitive activities. The findings reveal a differentiated and, to some extent, fragmented profile of cognitive development.

While pre-service teachers demonstrated moderate levels of cognitive awareness and cognitive flexibility, their mathematical thinking levels were found to be relatively high. This asymmetrical pattern suggests that mathematical thinking, as developed within current teacher education contexts, may not be sufficiently supported by corresponding cognitive and adaptive regulatory processes. In other words, pre-service teachers appear to be capable of engaging in mathematical problem-solving and knowledge construction, yet they may lack the cognitive depth required to systematically monitor, evaluate, and regulate these processes.

This finding, when considered alongside the discussion, points to a critical conceptual gap: the development of mathematical thinking does not necessarily guarantee the development of cognitive awareness and flexibility. Rather, these dimensions seem to evolve independently unless explicitly integrated within instructional and pedagogical frameworks.

Furthermore, the fact that the majority of participants perceived themselves as only partially competent in employing methodologies to enhance cognitive activities through educational resources highlights a structural limitation in teacher education programs. Despite the increasing availability of diverse educational resources—particularly in digital environments—pre-service teachers may lack the pedagogical knowledge and strategic competence required to utilize these resources in ways that effectively foster cognitive development.

Recommendations

Based on the findings of this study, the following recommendations are proposed to support the development of pre-service mathematics teachers' cognitive activities through educational resources.

From the professional development initiatives perspective; conferences, seminars, and workshops focusing on cognitive awareness, cognitive flexibility, and self-efficacy should be organized for pre-service mathematics teachers within faculties of education. These initiatives would help strengthen cognitive competencies and mathematical thinking skills in practical contexts.

From the curriculum enrichment and reform context; course curricula for pre-service mathematics teachers should be enriched with structured practices and activities explicitly aimed at enhancing cognitive competencies, including critical thinking, reflective learning, and creative problem-solving.

On the other side in order to effectively implement methodologies for improving cognitive activities, education faculties should consider renewing and adapting existing curricula to integrate educational resources systematically, both digital and traditional, into pre-service teacher training.

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Appendix

Semi-structured interview form

Semi-structured teacher interview form					
Part 1					
Your gender:	Female ( )		Male ( )		
Class you are studying:	1.Class ( )	2.Class ( )	3.Class ( )	4.Class ( )	
Part 2					

Evaluate your attitude towards your cognitive awareness level by ticking one of the 'Always', 'Sometimes' and 'Never' options (X).	Always	Sometimes	Never
While listening to the lecture, I question whether I understand			
I try to understand the purpose of the subject I will learn in the lesson.			
When I encounter a problem related to a subject that I do not know in the lesson, I anticipate possible solutions about that subject.			
Getting support from the teacher about the subject of the lesson helps me to be a successful student.			
I can identify critical concepts of the subject of the lesson			
I can evaluate myself whether the course has achieved its objectives or not.			
Evaluate your attitude towards your level of cognitive flexibility by ticking one of the 'Always', 'Sometimes' and 'Never' options (X).	Always	**Sometimes**	**Never**
I can express an idea in many different ways			
I can find useful solutions to problems that seem unsolvable.			
I am willing to produce creative solutions to problems.			
I am confident enough to try different behaviour patterns.			
In any situation that I encounter, I can act appropriately.			
Evaluate your attitude towards your level of mathematical thinking by ticking one of the options 'Always', 'Sometimes' and 'Never' (X).	**Always**	**Sometimes**	**Never**
My mathematical thinking skills are used in solving daily life problems.			
A good mathematician is one who has a high level of creative thinking.			
I solve problems in unconventional ways with my mathematical thinking skills			
With my mathematical thinking skills, I can connect with my old knowledge while constructing new knowledge.			

What are your competencies regarding your methodology for improving your cognitive activities through educational resources?

Very sufficient ( ) Sufficient ( ) Partially sufficient ( ) Insufficient ( ) Very insufficient ( )

Your feedback: ....